Exam Physics Laboratory 1: Data and error analysis **30 October 2013**

This exam consists of **5 exercises** on 2 pages. Make each exercise on a separate sheet of paper! Write your name and student number on each sheet of paper! Write clearly, using a pen (not a pencil).

Exercise 1 *(4 points)*

Rewrite the following results, using the correct notation:

a) $v = 2.71828$ m/s $\pm$ 2 mm/s

b) $L = 3.14$ km $\pm$ 1.5 cm

c) $C = 4722$ μF $\pm$ 0.42 mF

d) $R = 68$ MΩ $\pm$ 22 kΩ

Exercise 2 *(5 points)*

A resistor with resistance R carries a current I. The power P dissipated as heat by the resistor is given by $P = I^2R$. A resistor with $R = 330$ Ω is used, the accuracy of R is listed by the factory as 5%. The current is measured: $I = 0.28 \pm 0.01$ A.

Calculate the relative and absolute error in the power P and write the final result $P = ... \pm ...$ in the correct notation.

Exercise 3 *(5 points)*

Two independent measurements of the length L of a wire yield: $L_1 = 16.4 \pm 0.5$ m and $L_2 = 16.1 \pm 0.2$ m.

Calculate the weighted average length L and the error in L.

Exercise 4 *(10 points)*

The resistance R of an electrical circuit is measured 6 times, with the following results: $R = 47.1$ Ω, 47.4 Ω, 47.8 Ω, 46.9 Ω, 47.2 Ω, 47.5 Ω.
It is clear that the random error in R is much larger than the ± 0.1 Ω error of the measurement instrument.

a) Calculate the best estimate for the resistance of the circuit.

b) Calculate the best estimate for the standard deviation σ.

c) Calculate the error in the best estimate for the resistance calculated in part a).

d) How many extra measurements are needed to reduce the error calculated in part c) by a factor of 3?

e) Suppose the original experiment is repeated, again by measuring the resistance 6 times. What is the probability of finding a new result within the error limits calculated in part c)?

Please turn over for exercise 5.

x	$y \pm \Delta y$
1.00	10 ± 2
2.00	22 ± 2
3.00	32 ± 2
4.00	40 ± 2

Exercise 5 *(11 points)*

A series of 4 observations is given in the table above. The error in x is negligible. A straight line $y = ax + b$ is fitted to these observations. The following formulae are given:

$$a = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{N\sum x_i^2 - \left(\sum x_i\right)^2}, \qquad b = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{N\sum x_i^2 - \left(\sum x_i\right)^2},$$

$$(\Delta a)^2 = \left(\frac{1}{\sum x_i^2 - N\overline{x}^2}\right)\frac{\sum r_i^2}{N-2},$$

$$(\Delta b)^2 = \left(\frac{1}{N} + \frac{\overline{x}^2}{\sum x_i^2 - N\overline{x}^2}\right)\frac{\sum r_i^2}{N-2}.$$

a) Calculate the best estimate for a and b using the method of least squares.

b) Calculate the errors in a and b.

c) The student who has carried out the experiment wants to use the chi-square test to check whether the linear fit is acceptable. Calculate χ^2.

d) Suppose the 10% - 90% probability level is chosen. Using the table below, indicate whether the linear fit is acceptable or not.

e) Now assume $\Delta y = 0.5$ for all observations. Indicate whether the linear fit is acceptable or not for this case of smaller Δy.

$F =$ ν	0.01	0.10	0.50	0.90	0.99
1	0.000	0.016	0.455	2.706	6.635
2	0.020	0.211	1.386	4.605	9.210
3	0.115	0.584	2.366	6.251	11.35
4	0.297	1.064	3.357	7.779	13.28
5	0.554	1.610	4.351	9.236	15.09

Table 1: Cumulative χ^2 distribution $F(\chi^2|\nu)$.

Exam grade = (total of points) / 4 + 1.25